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DISTRIBUTION OF INCOME IN GREAT BRITAIN AND INCIDENCE OF INCOME TAX

SUMMARY

Mr. Mallock's estimates, 216. — Average number of incomes per unit of the range of incomes in each group used as a measure of density, 218. — Density coefficients, 220. — Three strata of income receivers, 221. — Density coefficients vary with the reciprocals of the cubes of the corresponding incomes, 222. — Corrected average incomes for groups, 224. — Rectified tables of income distribution, 226. — Advantage in taxing all incomes above £100, 227. — Formula for graduation of the tax rate, 228. — Tax yield controlled by varying the ultimate rate, 229. — Its incidence controlled by varying the size of income to which half the ultimate rate applies, 232. — Modifications to fit the use of step rates and of abatements, 236.

THE *Nineteenth Century* for March, 1910, contains a most interesting article by Mr. Mallock, in which he makes a careful estimate of the totals of the incomes of those in Great Britain receiving more than £115 per year.[1] Much in such estimates is to some extent conjectural, because the official statistics are still far from sufficient to furnish the full data required. Mr. Mallock's estimates have been objected to on this ground by statisticians whose professional practice compels them to keep rigidly within the limits over which the full facts are proven by well authenticated figures; but they appear to take into account more numerous considerations and detached items of reliable information than have been previously brought to bear upon this study; while his deductions from the available figures seem to be sane and prudent.

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[1] Since this paper was written, Mr. Mallock has published his estimates, more fully elaborated, in his volume on The Nation as a Business Firm, which is discussed by Professor Young below in the present issue, p. 376.

The table in which he summarizes these conclusions is the first seen by the present writer which bears internal evidence of enough completeness and approximate accuracy to make it worth while to found upon it any scientific theory or formula, rational or empirical. In any attempt to frame such a formula it should be constantly borne in mind that the formula can never be expected to fit exactly all the statistical figures throughout the full range of the phenomenon. This for two reasons: the first that the statistics are known to represent the actual facts only with rough approximation, and, as said above, to leave out of account many factors which can be filled in only by conjecture; and the second, that the actual facts themselves certainly do not follow with precision any exact mathematical law. As regards this latter reason, it is important to note that the approximation to mathematical law is certain to be closer the larger the number of individual cases which go to form the average. Thus there are enormous numbers of people who have small incomes and very few who have very large incomes. It follows that mathematical law should be more discoverable in the statistics of the poor. The number of the very rich is so small that no averaging proper is possible, and eccentric deviations from mathematical law are inevitable. Nevertheless, when the statistics of the very rich over a very long series of years become available, closer averages for this class also should become possible, provided that the concurrent influence of historical progress in time were known and allowed for.

The first four columns of Table I give the more important conclusions arrived at by Mr. Mallock. In the first column are given successive ranges of income from £115 upwards. In the second are given the aggregates of the incomes of all persons coming within these ranges. In the third appears the average individual income as estimated for each range, and in the fourth the number of persons receiving income within each range.

In the first place it must be noted that the figure in the third column is a purely conjectured average; secondly, that the number in the fourth column is an estimate obtained by dividing column 2 by column 3; thirdly, that this fourth column number has been revised by the present writer to give a closer arithmetic correspondence between it and columns 2 and 3 from which it is derived.

TABLE I

DISTRIBUTION OF INCOME

Range of Income	Total Income within Range	Assumed Average over Range	Number of Income Receivers within Range, deduced from Average	Number per £1 Range
£115– 150	Million £333	£145	2,295,000	65,600
150– 160	70	155	451,500	45,150
160– 200	250	180	1,389,000	34,725
200– 300	100	250	400,000	4,000
300– 400	66	350	188,500	1,885
400– 700	86	530	162,000	540
700– 2,000	207	1,200	172,500	133
2,000– 3,000	47	2,500	18,800	18.8
3,000– 5,000	54	4,000	13,500	6.75
5,000–10,000	67	7,000	9,600	1.92
10,000–50,000	40	19,000	2,100	.0525
50,000–———	35	110,000	320	......
Total	£1,355	......	5,102,820	......

Again, observe that the assumed averages are in many cases merely the arithmetic means between upper and lower limits of the range. It will appear presently that this arithmetic mean is always higher than the true average: consequently the numbers in column 4 are smaller than the actual numbers. In the first line of the table the error is greater, the assumed average being higher than the arithmetic mean.

From the table as it stands, in these first four columns no progressive law is apparent: the figures seem to vary in an extremely erratic fashion. This results, however, from the want of uniformity in the extent of the ranges in which the whole is sectioned. These ranges are £35, £10, £40, £100, £100, £300, £1300, £1000, £2000, £5000, £40,000, and from £50,000 upwards. Here there is neither uniformity nor any regular progression. Without reduction on some systematic plan, no law could be discoverable. Evidently the proper method is to reduce, if possible, to some one uniform range of income. This may be taken indifferently at £1000, £100, £10, or £1. If £1000 were taken, each result would be simply a hundred times greater than if £10 be used. In the poor end of the table, it is necessary to consider separately each £10 increment from say £100 up to £200, and if the number of income holders per £100 range were tabulated for this part of the table where the variation is so rapid, the figure could be understood only as ten times that for £10 range. On the other hand, at the other end of the table, the number per £100 range gives as result a small fraction of one individual, which again has no meaning except when understood to be simply the 100th part of the number per £10,000 range. Thus, if a £1 range be taken and the numbers in column 4 be divided by the corresponding range in pounds, we arrive at a series of figures giving a first approximation to the natural law of what may be called the Density of the Income Receiving Population at each level of income. The result of this division is shown in the fifth or last column of Table I, which has been added by the present writer.

The results have been plotted on a large-scale diagram as vertical ordinates against abscissae equaling the mean income over each range as given in Mr. Mallock's table. To the eye of the expert in diagram plotting the curve thus produced at once suggests a curve of reciprocals; not an hyperbola or curve of first power reciprocals, but one of reciprocals of either the square or the cube of the income. Employing another well-known form of logarithmic diagram, this same conclusion is confirmed.

Looking at the last column of this Table I, it is seen what an immense variation of this "income-density" exists. Between £115 and £150 it is 1¼ million times greater than between £10,000 and £50,000. If a natural law applies throughout, it can hardly be expected that the coefficients in the numerical expression of this law will be the same for all strata in the social aggregate.

To test the suggested law of the reciprocal of the cube of the income, each number in this last column is multiplied by the cube of the average income in the corresponding range. The resulting coefficients are placed below in

TABLE II

These density coefficients to be multiplied by 1000 million

Range £	115 150	150 160	160 200	200 300	300 400	400 700	700 2,000	2,000 3,000	3,000 5,000	5,000 10,000	10,000 50,000	50,000 ....
Density Coefficient	200	168	202	63	81	76	230	294	432	658	361	2,330

It will be perceived that this analysis divides the whole range very sharply into four classes: namely (1) below £200, (2) between £200 and £700, (3) between £700 and £50,000, and (4) above £50,000.

Within each class the coefficient varies irregularly, and not more than might be expected for reasons already mentioned, especially if it be remembered that the figures have been arrived at by the use of false average incomes applied to each range.

In the third class the coefficient increases somewhat rapidly from £700 up to £10,000, then falls equally rapidly. Above £50,000 in the fourth class it once more reaches a much higher figure than previously. From £700 upwards it is found, in fact, that the law of the inverse 2½th power, instead of the inverse cube, produces less variation of this density coefficient, examples being 607, 809, 362, and 504; but this variation is equally erratic in general character. The purpose of the present paper being to find a sure and fairly easy method of income-tax calculation, and the calculations upon the 2½th power basis being much more difficult than those on the 3d power basis, it has been decided to adhere to this latter and to sectionize the range above £700 into three instead of two classes.

The adoption of any one law enables one to calculate the corresponding true average according to this law over any given range. For instance, by a simple process of integration, the law of the inverse cube leads to the following value of the mean income between any lower limit I_1 of income and any other higher limit I_2; namely, mean $I_m = 2I_1I_2/(I_1 + I_2)$. If the aggregate of the incomes between these limits be known, the true value of the density coefficient for that range can also be directly calculated. This inverse-cube law gives the value

$$\text{Density Coefficient} = n = I_1 I_2 \,\Sigma I_{12}/(I_2 - I_1)$$

where ΣI_{12} means the above total of the incomes. This total multiplied by both upper and lower limits and divided by their difference (*i. e.* the range) gives the coefficient. It should here be specially noted that this coefficient is *not* the number of persons per £1 range: this number of persons equals the coefficient divided by the cube of the income.

In the following Table III the values of I_m and of n thus calculated from the totals given by Mr. Mallock are placed against each range in Mr. Mallock's table.

TABLE III

RECTIFIED AVERAGES AND DENSITY COEFFICIENTS

Range $I_1 - I_2$	Total ΣI_{12}	True Average I_m	Density Coefficient n divided by One Thousand Million	Number of Income Receivers
£115– 150	Million £333	£130.2	164	2,558,000
150– 160	70	154.8	168	452,000
160– 200	250	177.8	200	1,406,000
200– 300	100	240.0	60	416,700
300– 400	66	342.8	79	192,500
400– 700	86	509.1	80	168,900
700– 2,000	207	1,037	223	199,600
2,000– 3,000	47	2,400	282	19,580
3,000– 5,000	54	3,750	405	14,400
5,000–10,000	67	6,667	670	10,050
10,000–50,000	40	16,667	500	2,400
50,000– 10^7	35	99,500	1760	352
Total	£1,355	. . . . . .	. . . . . .	5,440,500

The number of receivers of income as re-calculated from the corrected average is placed in the last column. These more correct averages being lower than those assumed by Mr. Mallock, the number of such receivers accounted for becomes greater, the total being 330,000 more than in Table I.

It will be noted that the variation of the density coefficient is now much more regular.

The most marked features are the sudden great drop at the limit £200; the equally sudden rise again at £700; and the further great sudden rise above £50,000.

There is little variation from £700 to £3000 ; and again no extreme variation between £3000 and £50,000, there being in this range a moderate rise followed by a fall.

The limits £200, £700, £3000, and £50,000 are therefore taken in what follows as dividing the whole range into five classes. The next table shows the coefficients worked out for these classes from the total incomes included within them.

TABLE IV

DISTRIBUTION OF INCOME IN FIVE CLASSES

Range $I_1 - I_2$	Total ΣI_{12}	Average I_m	Density Coefficient $\frac{n}{10^9}$	Number of Income Receivers
	Million			
£115– 200	£ 653	£146	176.7	4,473,000
200– 700	252	311	70.56	810,000
700– 3,000	254	1,135	231.9	223,800
3,000–50,000	161	5,660	513.9	28,440
50,000– 10^7	35	99,500	1,760	350
Total	£1,355	......	....	5,535,590

This analysis makes the number of persons receiving incomes over £115 a little higher, namely 95,000 more, than the last more detailed analysis. This is due to the shifting of the positions of the averages employed in the calculations.

It may here be observed that the above 5½ million income receivers are generally reckoned to correspond to five times as many men, women, and children, or 27,678,000, since five is assumed as the average size of a family.

Mr. Mallock's figures go as low as £115 only because he fixed upon this limit as that below which all persons should be completely exempt from income tax. It is impossible that the law of inverse cubes should hold down to the lowest standards of living. It would mean the existence of innumerable hordes of half- or three-quarters destitute men. Evidently another forcible law comes into play at these low levels; namely, that it is impossible to live inside a not altogether savage human community on less than certain minima of means. Here it may be remarked that a mathematical "frequency" or density law which would more or less accurately cover the whole range including the lowest strata of destitution might be easily discoverable if the statistical data for it existed. But for the special purpose of the present article this complete law is unnecessary as it is impossible or at least undesirable to tax these lowest strata.

It may, however, be taken for granted that the same law as applies between £200 and £115 applies down to £100. This is the limit below which the present paper proposes to exempt every one from all taxation both direct and indirect. We round off the density coefficients in the direction of safety in estimating for income tax, and take the upper limit as ten million pounds. The substitution of this upper limit for infinity, or any figure greater than ten million, makes no material difference in the calculated result.

These adjustments give the following classification of the taxable income of the nation set forth in Table V.

TABLE V

TAXABLE INCOME OF THE NATION

Range	Total Income	Average Income	Density Coefficient 10^9	Number of Taxable Persons
	Million			
£100– 200	£880	£133⅓	176	6,600,000
200– 700	250	311⅑	70	804,000
700– 3,000	252	1,135	230	222,000
3,000–50,000	160	5,660	510	28,270
50,000– 10^7	35	99,500	1,760	350
Total	£1,577	......	......	7,654,620

This number of persons, reckoning each to represent a family of five accounts for 38¼ million of the population, leaving some five million, or one million families, with less than £100 income. This closely accords with the estimate generally made, the income being that of the whole family regularly and irregularly earned.

Before applying these data to an estimate of possible income tax revenue, it will be interesting to tabulate in greater detail this law of distribution of wealth. This is done in the following table.

TABLE VI

DISTRIBUTION OF INCOME

Range	Average	Number of Families per £1 Range	Aggregate Income per £1 Range	Range	Average	Number of Families per £1 Range	Aggregate Income per £1 Range
£100– 120	£109.1	134,420	14,666,667	£1,000– 1,500	£1,200	127.8	153,333
120– 140	129.2	81,090	10,476,190	1,500– 2,000	1,714	44.73	76,667
140– 160	149.3	52,625	7,857,143	2,000– 2,500	2,222	20.70	46,000
160– 180	169.4	36,070	6,111,111	2,500– 3,000	2,727	11.25	30,667
180– 200	189.4	25,810	4,888,889				
				3,000– 4,000	3,428	12.40	42,500
200– 250	222.2	6,301	1,400,000	4,000– 5,000	4,444	5.74	25,500
250– 300	272.7	3,422	933,333	5,000– 6,000	5,454	3.117	17,000
300– 350	323.1	2,063	666,667	6,000– 7,000	6,461	1.880	12,143
350– 400	373.3	1,339	500,000	7,000– 8,000	7,466	1.220	9,107
400– 450	423.6	918	388,889	8,000– 9,000	8,471	0.836	7,083
450– 500	473.7	657	311,111	9,000– 10,000	9,473	0.598	5,667
500– 550	523.8	486	254,545				
550– 600	573.9	369.6	212,121	10,000– 20,000	13,333	0.1913	2,550
600– 650	624.0	287.6	179,487	20,000– 30,000	24,000	0.0354	850
650– 700	674.1	228.3	153,846	30,000– 40,000	34,286	0.0124	425
				40,000– 50,000	44,444	0.00574	255
700– 800	746.6	550.2	410,714				
800– 900	847.1	377.1	319,444	50,000– 100,000	66,667	0.00528	352
900–1,000	947.3	259.1	245,454	100,000– 200,000	133,333	0.00066	88
				200,000– 300,000	240,000	0.0001222	29⅓
				300,000– 400,000	342,857	0.0000428	14⅔
				400,000– 500,000	444,444	0.0000200	8.8
				500,000– 1,000,000	666,667	0.00000528	3.52
				1,000,000–10,000,000	1,818,181	0.00000097	0.176

This table demonstrates very clearly the importance in the levy of taxation of going to the lowest level that may be deemed just, generous, and economically practicable. Because the £20 range from £100 to £120 yields 1½ times as much aggregate income as the £20 range from £120 to £140; twice as much as the same range from £140 to £160; and thrice as much as the same between £180 and £200.

Remembering that an "average family" consists of two adults and three children ranging in age up to fifteen or sixteen, the following is an estimate of the minimum cost of living a physically and morally healthy life. Food, £54; clothing, £14; coal and firewood and gas, £6; rent, rates, and water, £15; furniture and household utensils, £3; holidays and amusements, £4; sickness, accident, and old age insurance, £4; total, £100.

This does not include the cost of schooling, now undertaken by the State and paid for out of rates and taxes; nor does it include the cost of any other public services, such as sanitation, roads, police, etc., except as included in rates. It includes a small item for sickness because hospitals are as yet only to a very small extent supported out of taxation and are largely financed by the voluntary contributions of the artisan classes; also a small item for old age because no one supposes that the present government old age pension (five shillings) is sufficient for the support of aged parents.

The taxation proposal now to be made is of the "free breakfast table" kind. It is clear that for the physical and moral welfare of the nation, it is not advisable to lay any compulsory taxes upon those whose income per family of five is below £100: especially when, as now, the term "taxation" is taken as excluding "rates." So long as "indirect" taxation is maintained as a means of compelling the poor to contribute to state revenue, it may be easily demonstrable that it is most economically levied upon tea, coffee, and sugar. But for that very reason, it seems clear that indirect taxation ought not to be relied upon for this purpose, because it is undeniably both unjust and cruel to increase forcibly the prices of what are real necessaries of modern life to the women and children of the very poor. There is no such argument for abandoning indirect taxation of the luxuries of either poor or rich, such as wines, spirits, and tobacco.

Excluding these taxes upon luxuries, and the stamp and other like duties, the remainder of our present indirect taxation yields a revenue of roughly thirty million pounds, and the income tax roughly another thirty million. It is proposed to raise the whole of these sixty millions by a graduated income tax upon those having over £100 income per family. To go below this limit interferes with the possibility of physical and moral welfare, and is therefore unjust and inexpedient. So far as those with less than this income insist upon spending part of it upon taxed luxuries, the influence of taxation in restricting such expenditure is both just and also very desirable.

A practically perfect graduation of income tax may be expressed algebraically by the formula

$$\text{Rate of Tax} = \frac{M \times \mathit{Income}}{\mathit{Half\ Limit\ plus\ Income}}$$

$$\text{or } i = \frac{MI}{H + I};$$

where what is meant by H, or "Half Limit," is that income at which the rate becomes *half the ultimate* *rate which is not quite reached even with multi-millionaires,* and where M means *this ultimate rate slightly above that imposed upon multi-millionaires.*

If the Half Limit be placed at £3000, Table V shows that this is just at the top of what may be called the rich middle class. It is not, of course, pretended that there are any perfectly sharp lines of demarcation between the five classes of Table V. Probably they overlap each other considerably. But the strikingly large differences discovered in the "density-coefficients" proves the real existence of six main classes in our society: (1) the very poor under £100; (2) the " artisan " up to £200; (3) the poor middle class up to £700; (4) the rich middle class up to £3000; (5) the very rich up to £50,000; and (6) the millionaires, so called because each enjoys as income a considerable fraction of a million pounds, their average being one tenth of one million. The Half Limit £3000 makes the rate of tax for £750 equal to 750/3750 = $\frac{1}{5}$ of the ultimate rate; so that, for example, if the ultimate rate M = 25 per cent or 5*s.* per £1, that for £750 would be 1*s.* This £750 is just well clear of the somewhat unfortunate lower middle class who reach up to £700, and who cannot avoid many social expenses rather disproportionate to their means.

The ultimate rate M has to be fixed from year to year by the Chancellor of the Exchequer in proportion to the needs of the revenue. It can be varied in more or less exact proportion to these needs. The revenue levied upon each class and upon each individual varies with M in the same proportion for all; and the total state revenue yielded from income tax varies in the same proportion.

It has already been noted that ordinary integration from the law of inverse cubes gives the aggregate income between the limits I_1 and I_2 equal to $\Sigma I_{12} = n(I_2 - I_1)/I_1I_2$ for a constant value of the density coefficient n between these limits. For the whole of any one class throughout which a uniform value of n rules, a similar integration shows that the application of an income tax graduated in the above explained manner yields a revenue,[1]

Income tax revenue between limits I_1 and I_2 equals:

$$2.3026 \frac{Mn}{H} \log_{10} \left\{ \frac{(H + I_1) I_2}{(H + I_2) I_1} \right\}.$$

Table VII contains the result of applying this formula to the five classes already particularized for two examples of the value of M, namely twenty and twenty-five per cent.

TABLE VII

GRADUATED INCOME TAX REVENUE, WITH ULTIMATE RATE 20 AND 25 PER CENT.

Class Limits of Income	Density Coefficient 10^9	Aggregate Class Income	Income Tax Revenue $M = 20\%$	Income Tax Revenue $M = 25\%$
		Million		
£100- 200	176	£880	£7,762,000	£9,702,000
200- 700	70	250	5,206,000	6,507,000
700- 3,000	230	252	14,902,000	18,628,000
3,000-50,000	510	160	21,586,000	26,983,000
50,000- 10^7	1,760	35	6,802,000	8,503,000
Total	......	£$1,577 \times 10^6$	£56,258,000	£70,323,000

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[1] The factor 2.3026 appears here in consequence of the formula being adapted for use with common decimal logarithms instead of "natural" or "Naperian" logarithms. With H = £3000 and M = 25 per cent this becomes

$$\frac{0.1919}{1000} n \log_{10} \left\{ \frac{(3000 + I_1)I_2}{(3000 + I_2)I_1} \right\}.$$

The value of the Ultimate Rate M which would yield approximately sixty millions revenue is 21.33 per cent. The average rate upon 1577 million pounds that would yield sixty millions is 3.8 per cent.

If the Half Limit H were made less than £3000, the general effect would be to levy a proportionately larger share of the total burden from the poorer classes. To appreciate in more precise manner the effect upon Total Income Tax Revenue of varying H, the simplest theoretic possibility may be assumed by way of illustration: namely, that the whole range from £100 to ten million pounds is governed by one and the same uniform density coefficient. If then we use Q as contraction for

$$\frac{2.3026}{H}\log_{10}\left\{\frac{H+100}{H+10^7}.10^5\right\} = Q$$

$$= \text{very nearly } \frac{2.3026}{H}\log_{10}\left(\frac{H}{100}+1\right).$$

We have

$$\text{Aggregate Income Tax Revenue} = M\,n\,Q.$$

The following Table VIII gives the values of the function Q between these limits 100 and ten million for values of H up to 10,000. For arithmetical convenience the values of Q are given as multiplied by one million.

TABLE VIII

AGGREGATE INCOME TAX REVENUE = $M\,n\,Q$

£ H	1,000	2,000	3,000	4,000	5,000	6,000	8,000	10,000
$Q \times 10^6$	2,397.9	1,522.0	1,144.5	928.3	786.2	685.0	549.2	461.4

To give the aggregate income of 1355 million pounds between limits £115 and ten million pounds as found by Mr. Mallock, the appropriate uniform Density Coefficient is 155,800 million. This multiplied by the above gives

TABLE IX

AGGREGATE INCOME TAX REVENUE FOR UNIFORM $n = 1558 \times 10^8$. MILLION £S TO BE MULTIPLIED BY ULTIMATE RATE M

H £	1,000	2,000	3,000	4,000	5,000	6,000	8,000	10,000
$nQ \div 10^6$	373.6	237.1	178.3	144.6	122.5	106.7	85.57	71.88

Thus if the Ultimate Rate were 20 per cent, the total yield would be 74.72 million pounds with Half Limit £1000; but only 35.66 million, or less than half as much, with Half Limit placed at £3000; and no more than 24½ million if it were placed at £5000.

This table is given only to illustrate the broad effect of changing the Half Limit. We have found above that the Density Coefficient is actually very far from being constant, and its changes from class to class result in very materially different proportions than those shown here in Table IX.

To show how the rate of the tax imposed increases with the income in the most general manner the following Table X is given, where the ratio of the tax-rate on any income to the Ultimate Rate is coördinated with the ratio of the corresponding income to the Half Limit income. That is, i/M is coördinated with I/H.

TABLE X

i/M	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13		
I/H	.0309	.0416	.0526	.0638	.0753	.0870	.0990	.1111	.1236	.1363	.1495		
i/M	.14	.15	.16	.17	.18	.19	.2	.21	.22	.23	.24		
I/H	.1628	.1764	.1904	.2045	.2195	.2346	.2500	.2658	.2821	.2987	.3158		
i/M	.25	.275	.3	.325	.35	.375	.4	.425	.45	.475	.5		
I/H	.3333	.3793	.4286	.4815	.5385	.6	.6667	.7391	.8182	.9048	1		
i/M	.525	.55	.575	.6	.625	.65	.675	.7	.725	.75			
I/H	1.105	1.222	1.353	1.5	1.667	1.857	2.077	2.333	2.636	3			
i/M	.775	.8	.825	.85	.875	.9	.925	.95	.975	.98	.985	.99	.995
I/H	3.444	4	4.714	5.667	7	9	12⅓	19	39	49	65⅔	99	199

To illustrate this graduation further, the next Table XI gives its results if the Half Limit be £3000 and the Ultimate Rate 20 per cent.

TABLE XI

H = £3000 : M = .2

I	100	120	140	160	180	200	250	300	400	500
i	.0064	.0077	.0089	.0101	.0113	.0125	.0154	.0182	.0235	.0286
I	600	700	800	1,000	1,250	1,500	1,750	2,000		
i	.0333	.0378	.0421	.0500	.0588	.0667	.0716	.0800		
I	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000		
i	.1000	.1143	.1250	.1330	.1400	.1454	.1500	.1538		
I	15,000	20,000	30,000	40,000	50,000	100,000	200,000	500,000		
i	.1667	.1739	.1818	.1860	.1887	.1942	.1970	.1988		

If the Half Limit H were well chosen at any one date, there would be no rational ground for changing it for a long period; not until the general condition of society and the financial relations between the various classes of society had undergone substantial change.

It is nevertheless of interest to inquire what effect change in H has upon an individual income tax at any given level of income. From the remarks already made it is clear that lessening H means a larger total revenue from taxes with any given ultimate rate M, and also means raising the ratio of the tax-rate on any income I to this Ultimate Rate M throughout the whole scale of I. But it raises it in different proportions at different parts of the scale; relatively less at the top and relatively less at the bottom. And since only a definite prescribed revenue is required, such lowering of H would permit of lowering M also. Does the decrease of M compensate for the increase of proportion of i to M; or when does it do so and when the reverse ?

Let the prescribed revenue required be called R. Then the formula already given shows that the needful Ultimate Rate $M = R/nQ$. Now on any income I the tax is levied at the rate

$$i = M\Big/\left(1+\frac{H}{I}\right) = \frac{R}{n}\Big/\left(Q+\frac{QH}{I}\right).$$

Reference to Table VIII shows that increase of H decreases Q but increases QH. If the decrease of Q be greater than the increase of QH/I, then the increase of H will also increase i. Now the increase of QH/I is inversely proportional to I. So that for I greater than a certain limit, the negative change in Q must be greater than the positive change in QH/I, and the effect is to increase i. For incomes below this limit increase in H will decrease i.

This result is worthy of careful observation. A numerical illustration will make it clearer. Compare

H = 1000 with H = 2000 at the two income levels £500 and £2000. Taking the figures for Q from Table VIII, we find:—

For I = £500 with H = 1,000; $i = \frac{R}{n}/(2{,}398 + 4{,}796)10^6 = \frac{R}{n}/7{,}194 \times 10^6$;

For I = £500 with H = 2,000; $i = \frac{R}{n}/(1{,}522 + 6{,}088)10^6 = \frac{R}{n}/7{,}610 \times 10^6$;

For I = £2,000 with H = 1,000; $i = \frac{R}{n}/(2{,}398 + 1{,}199)10^6 = \frac{R}{n}/3{,}597 \times 10^6$;

For I = £2,000 with H = 2,000; $i = \frac{R}{n}/(1{,}522 + 1{,}522)10^6 = \frac{R}{n}/3{,}044 \times 10^6$.

At the lower income £500 the change from Half Limit £1000 to £2000 has decreased i by 5½ per cent. At the higher income £2000 the same change has increased i by 11.8 per cent.

By applying the ordinary rules of differentiation to this problem, it may be proved that increase of H produces increase of i when I is so great as to make

$$H/I \text{ less than } \left\{\left(1+\frac{100}{H}\right)\log\left(1+\frac{H}{100}\right)-1\right\}.$$

Here the natural or Naperian logarithm is used, and, if the decimal logarithm be used, then the logarithm on the right-hand side of the equation must be multiplied by 2.3.

For all incomes less than this limit, the rate i is lessened by increase of H.

Take as example H = £3000; then the right-hand side of the above equals $1\frac{1}{30} \log 31 - 1 = 2.548$, and for incomes greater than 3,000/2.548 = £1177, an increase of H above £3000 increases the income tax, while for incomes less than this such increase of H diminishes the tax on individuals. These limits are for

H = £	1,000	2,000	3,000	4,000	5,000	6,000	8,000	10,000
Limit = £	650	910	1,177	1,425	1,661	1,887	2,319	2,731

One other point as regards graduation of income tax is worthy of careful consideration. At present the successively higher rates rise by sudden steps of large magnitude. The result is that anywhere near each limit where the rate steps upwards there is an almost irresistible temptation to understate one's income, because the man just £1 under the limit is very largely better off — is left with a larger remainder after payment of the tax — than he who is just £1 over the limit. This is a ridiculously unjust result. Probably a stepped schedule of rates is a practical necessity for the revenue office. It should be accompanied by a schedule of steps of income, differences less than such steps of income being ignored by the surveyors: an excess of income above each limit less than the step being exempt from tax.

If at any limit in such a stepped schedule the tax rate be i and the succeeding step be Δi; that is, the next scheduled rate be $i + \Delta i$; while the income is I and the income step not subject to tax be ΔI; then those that pay on an income $I + \Delta I$ at the rate $i + \Delta i$, will be left no worse off than those that pay on I at the rate i, if $\{I + \Delta I\} \{1 - i - \Delta i\}$ be greater than $I(1 - i)$. Omitting minute quantities, this means that the income increment which should be ignored in respect of tax, ought to be

$$\Delta I \text{ greater than } \frac{I}{1-i} . \Delta i.$$

The steps Δi in the rate should be as small as may be considered convenient for revenue office calculation, and the steps of ignored income adjusted according to this formula.

Thus if on income £750 the tax be 5 per cent, and if the next higher scheduled rate be .052; then $\Delta i = .002$, and on $\Delta 1 = \frac{750}{.95} \times .002 = £1.58 =$ say £1, 12*s.* 0*d.* no tax should be charged. But if the rate were stepped from .05 to .055, then $\frac{750}{.95} \times .005 = £3.95$, or say £4, should be exempt.

Again if at $I = £4{,}500$, the rate be $i = .15$ and the next higher rate be .152, then any excess above £4500 up to $\Delta I = \frac{4500}{.85} \times .002 = £10, 12s.\, 0d.$ should be exempt. That is to say that the man whose income is £4510 should be taxed at the rate .152 upon £4500 only.

Many persons prefer the system of " abatements " or " allowances " upon the income taxed rather than graduation of rate of tax. Any continuous formula for graduation of rate may be alternatively expressed as one for " abatement " of income to be taxed at a uniform rate. The rule here recommended, namely, Rate of Tax equals $MI/(H+I)$ is easily seen to be equivalent to an abatement equal to $HI/(H+I)$ on the income I, the remainder to be taxed at the full rate M. This is perhaps a more soothing manner of expressing the formula, especially as the magnitude of the abatement always rises with the income, instead of decreasing as our present clumsy schedules of abatements make it. But it rises, of course, at a decreasing rate: the rate at which it rises with the income itself is the square of $H/(H+I)$. The amount of the tax to be paid on any income I is $MI\left(1 - \frac{H}{H+I}\right)\cdot$ When the income equals the " Half Limit," the abatement equals half the income.

Finally it may be pointed out that if, in order to encourage the now growingly unpopular institution of marriage, the total income of a man and his wife were to be reckoned for taxation purposes as two equal incomes of two persons, then the limit below which incomes were exempt from taxation would need to be reduced from £100 to £50 and besides this either *M* would need to be raised or *H* diminished in order to realize the same total revenue.

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